NETHERLANDS CENTRAL BUREAU OF STATISTICS

Department of Statistical Methods
P.O. Box 959, 2270 AZ VOORBURG, The Netherlands

EXCESS DEMAND IN THE KELLER MODEL

O.J.C. Cornielje*)
C. Zeelenberg**)

* Faculteit der Economische Wetenschappen en Econometrie, Vrije Universiteit Amsterdam
** Department of Statistical Methods, Netherlands Central Bureau of Statistics
The views expressed in this paper are those of the authors and do not necessaril reflect the policies of the Netherlands Central Bureau of Statistics.

BPA no.:16015-89-M1
December 7, 1989

Proj.M1-81-208
First version

Summary

In this paper the Keller (1980)-framework for an applied general equilibrium model is adapted to allow for rationing of households. To accomplish this, the concept of virtual prices (Neary and Roberts, 1980) is used. Due to this adaption, the calibration of this type of linearized models to a benchmark-year data-set, must be adapted too. After outlining these changes, the adapted calibration-method is applied to a Dutch historical time-series. Afterwards, the modified framework is used for a disaggregated 114-sector model of The Netherlands in 1981. The results show that this adaption to allow for rationing is quite important for the model outcomes.

EXCESS DEMAND IN THE KELLER MODEL

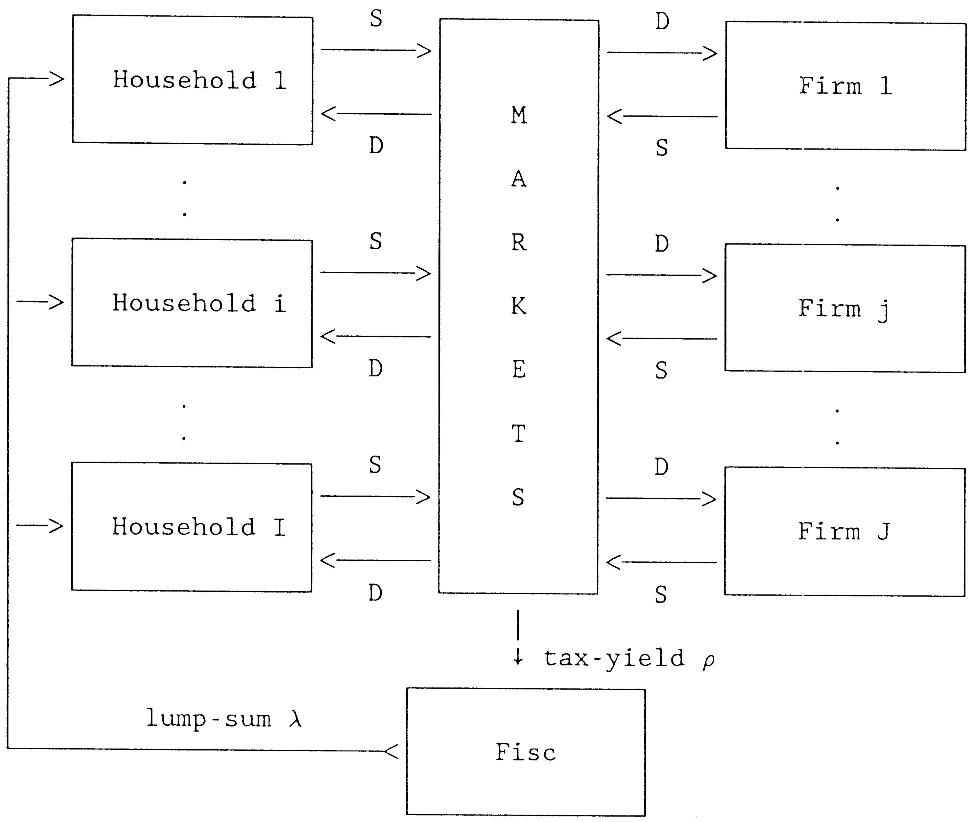
1. Introduction

In this paper some enhancements to a particular Applied General Equilibrium model are presented. The framework of this model was developed by Keller (1980), and originally used for a model of the Dutch economy of 1973. Although this model is constructed according to specific assumptions about the behaviour of the various economic agents, the enhancements are based upon general mechanisms which can be used in other Applied General Equilibrium models as well. Here, the Keller-model is expanded to allow for price-rigidities and rationing of households, following Cornielje and Keller (1983), Cornielje (1985) and Cornielje and Van der Laan (1986).

In the remainder of this section the skeleton of the Keller-model is outlined. Section 2 presents a summary of the theoretical backgrounds of household rationing. The construction of a benchmark-year data set under rationing is studied. Section 3 presents the application of the methods developed in section 2 to a time-series of 'Total Accounts' as presented in Cornielje (1989). In section 4 an application to the 114-sector Keller-model for The Netherlands, 1981, is presented. This latter model has first been used in Keller e.a., 1988. In an appendix the computational changes to the original framework in Keller (1980) are sketched.

In figure 1 the components of the Keller model are sketched. The Keller model describes an economy as a set of three types of agents, consumers (households), producers (firms) and a fisc. Consumers and producers demand and supply goods, which are both commodities and factors of production. The goods are sold and bought on markets. Each agent faces net (effective) prices, including transaction taxes, when buying or selling goods. These prices differ from the market prices due to taxes. The tax yields are collected by the fisc. The fisc redistributes this tax yield to the consumers as lump-sum income. Total income of a consumer consists of this lump-sum income and the yield of the goods sold. For all actors, total income equals total expenditures by definition.

Figure 1 *The skeleton of the Keller model*



The Keller-model describes a static equilibrium, which can be altered by changes in the exogenous tax rates. Intertemporal effects are not taken into account, except for savings and the production of capital goods.

The original model assumes full competition on all markets. Demand and supply is equalized at all markets by instantaneous adaption of all prices and the total tax-yield.

Consumers optimize their utility function with respect to their budget constraint. Producers optimize their profit with respect to the technological production constraint. As a result of the assumption of free entry and exit of firms, excess profit is zero and the production constraints show constant returns to scale. Consumers and producers face nested Constant Elasticity of Substitution utility and production functions (see Keller, 1976). These functions may be considered as local approximations to an unspecified global utility or production function. Non-unitary income elasticities of consumers are allowed for by a shift of the origin of the commodity space. The parameters of individual behaviour are determined by 'guestimation', i.e. inspection of the literature on consumer economics, and by the cost and income shares of respectively firms and households. Cost and income shares are derived from the so called Total Accounts, which are a variant of the Social Accounting Matrices. These Accounts are also used for determination of the average tax rates. Marginal tax rates may differ from average rates. Then, these marginal rates have to be determined from study of tax laws.

Savings are modeled as the demand for capital goods. Capital goods are the only source of wealth which can be transferred to the next period. Capital goods are produced by one artificial firm sector, demanding the investments done by other firms. Savings are a constant fraction of household income. As this constant fraction of income may differ among households, savings are modeled in a Kaldorian style. Except for 'replacements' due to depreciation, and wear and tear, investments are done by households. Therefore, firm savings by retained earnings are fully ascribed to household sectors, while replacements are modeled by demand for capital goods of firms.

Investments do not increase the capital stock in the period under consideration. Therefore, the distribution of the investments to the various production sectors can be discarded of. The existing capital stock is perfectly mobile. The usage of the existing capital stock is modeled as the demand and supply of 'capital services'. The market price of these capital services is equalized among firms due to the perfect mobility.

No money occurs. The actors do not experience some form of 'money illusion'. Normalization of prices takes place by usage of some basket of goods. This basket is chosen such that the lump-sum redistribution by the fisc is independent of changes in this price-index.

The foreign sector and public sector are modeled as distinct household or production sectors. The foreign sector is represented as a household which shows a perfect elastic reaction to non-proportional price changes of the goods exported from the country under consideration as result of the 'small open country' assumption. The price structure of imports is fixed and therefore imports can be considered as one good, according to Hicks' composite commodity theorem (see Hicks, 1946). Imports compete directly with domesticly produced goods. The demand for imports is modeled as demand for the output of firms, while firms demand the imports as part of their input into the production process. In the production process imports and domestic production are aggregated into one output, which is demanded by other firms and the households. Thus, the Armington assumption, that imports and exports are distinct goods, holds.

The public sector is modeled as a household, which demands various inputs to 'produce' public welfare. Alternatively, the public sector is split into one firm producing public goods, and one household, demanding these public goods. Its demand for the various goods is fixed according to a Leontief utility function. When policy measures are compared, public budget is balanced to retain a constant public utility level. If one assumes a private utility function, which is separable between the utility derived from the consumption of the public good and the utility derived from the private goods consumption, then the change in the latter represents the change in total utility if public utility is kept constant. Thus, the concept of differential incidence is used.

The model is solved by log-linearizing all behaviourial relations and equilibrium conditions. Therefore, the results are valid for small changes of the exogenous policy variables only.

The changes of individual welfare are measured by the compensating variation expressed as a percentage of sectoral or national income. The impact of various tax measures can be normalized such that the welfare of the public household increases by 1% of its income. This facilitates the study of the differential budget incidence.

2. Rationing by virtual taxes

The method to introduce rationing in the model uses the concept of 'virtual taxation' or 'virtual prices'. This concept is based on Neary and Roberts (1980), who draw on the paper of Rothbard (1941). The method is also used in Grais e.a (1986) for Turkey. Cornielje and Keller (1983) first applied this method to the Keller model for the Netherlands in 1973 but had to make some approximations about the second order effects of redistribution of the virtual tax yield. In this paper the method is incorporated in the Keller model using the linearized nature of the model. Cornielje and Van der Laan

(1986) have proven that a virtual tax equilibrium is equivalent with a Drèze-type equilibrium with price-rigidities for exchange economies.

In this section the theory of representing rationing by virtual taxation is summarized shortly. Then, the adaptions necessary to the benchmark year data-set are explored. Initially rationing occurs. Therefore, the absolute level of virtual taxation must be determined and incorporated into the initial equilibrium. The calibration method of matching a mathemetical framework of an applied general equilibrium model to a benchmark year data-set must be altered to provide for the additional information about excess demands. The results as presented in Cornielje (1985) are repeated shortly. There, a method is derived by which it is possible to calibrate a rationed household to a given observation of its trade and excess demand. In Mansur and Whalley (1984), calibration of a general equilibrium model is decomposed into smaller problems of calibrating individual agents, each of which can be handled separately. They provide a simple example for a household with a single level CES utility function and unitary income elasticities. Its demand is matched to trade as observed in a so-called benchmark-year data set, using a pre-determined estimate of the Allen elasticity of substitution. This example is extended in Cornielje (1984) to generalized multi-level CES utility functions as proposed by Keller (1976). These functions allow for non-unitary income elasticities too. There, besides the knot-structure of the utility-function, and accompanying elasticities of substitution, pre-determined income elasticities have to be known in order to calibrate the utility function.

In case of disequilibria on some markets, thus of rationed agents, the proposed methods in Mansur and Whalley (1984) and Cornielje (1984) are inappropriate because it is assumed that actual trade is equal to notional demand and supply. In this section an adaption of the calibration method is derived for such situations.

We suppose that only observations are given about real expenditure flows, relative prices paid by each agent and the magnitude of the excess demands. Then we show how rationing can be represented by virtual prices and income compensations and how these prices can be determined given observed excess demands and known demand functions. Next, we combine this idea of representing rationing by virtual taxes and calibration into one method to determine some unknown parameters of the demand functions and virtual prices simultaneously, using the observations as given above.

First, we introduce some terminology and the concept of virtual prices. From now on we will speak of demand only and denote supply by negative demand. Let p_n be the observed price of a good $n = 1,\dots,N$, p_n^* its virtual price depending on the actual trade z_k, $k = 1,\dots,N$ and $\lambda_n^* = (p_n^* - p_n)z_n$ the virtual lump-sum transfer which accompanies the imposition of the virtual tax at rate $t_n^* = (p_n^* - p_n)/p_n$. Let $g_n(p_k,\ k = 1,\dots,N;\ \lambda|\beta)$, $n = 1,\dots,N$ be the unconstrained Marshallian demand functions, where λ is lump-sum income and β is a vector of parameters. Then we define for each good $n = 1,\dots,N$ *notional demand* by

$$q_n = g_n(\ p_k\ ,\ k = 1,\dots,N;\ \lambda\ |\ \beta\ ) \quad , \tag{1}$$

effective demand by

$$q_n'' = g_n\left(p_n ; p_k^*,\ k<>n,\ k = 1,\dots,N;\ \lambda + \sum_{k\neq n} \lambda_k^* \,|\, \beta\right) \quad , \tag{2}$$

and *constrained demand* by

$$q_n^* = g_n\left(p_k^*,\ k=1,\dots,N;\ \lambda + \sum_{k=1}^{N} \lambda_k^* \ |\ \beta\right) \quad , \tag{3}$$

where $q_n^* = z_n$, and, therefore, p_n^*, $n = 1,\dots,N$ is, in fact, the solution of the last set of equations. The definition of effective demand corresponds to the one given by Benassy (1975) where effective demand is defined to be the demand on a market taking into account the rationing constraints at other markets only.

Suppose observations are available for q_n'', z_n, p_n, $n = 1,\dots,N$ and λ for, say, one benchmark year. Then p_n^*, $n = 1,\dots,N$ can be solved from the set of constrained demand equations (3) if the functional form $g_n(.)$ and the vector β are known. Under conditions given by Neary and Roberts (1980) such a solution exists; in particular if $g_n^* = z_n$ for a good n then $p_n^* = p_n$.

However, the observations of the excess demands $q_n'' - z_n$, $n = 1,\ldots,N$ are not used. Thus, evaluating these excess demands from the computed virtual prices may result in different values with respect to the observed ones. The core problem of calibration is to match demand functions exactly to observed demands by determining not yet specified parameters in the demand system. So, suppose that besides the virtual prices p_n^*, $n = 1,\ldots,N$ the N-1 vector β is not yet known. Then calibration under rationing is equal to solving the sets of equations (2) and (3) simultaneously for β and p_n^*, $n = 1,\ldots,N$, where the left-hand sides are observed. At most N-1 parameters and N-1 virtual prices can be determined as only N-1 independent equations are given by (3) as result of Walras' law, and as only N-1 equations are given by (2) because N-1 markets can be in disequilibrium simultaneously.

As no explicit solution is available in general, we propose to solve (2) and (3) iteratively, by solving in each iteration β from (3) for a given set of virtual prices and these prices from (2) for the determined β, and using these new values in the next iteration, etc., until the process converges. If no rationing occurs the process stops after one iteration as $p_n^* = p_n$, $n = 1,\ldots,N$. This turns out to be the original calibration method with an additional check on the outcomes. Therefore the method proposed can be seen as a direct extension of the normal calibration method to one suitable for disequilibrium situations.

When applied to the Keller model, an important implication of the method proposed is that it assumes that the local behaviour as described by the marginal income elasticities and the substitution elasticities of a multi-level CES-function determines global behaviour as described by global CES-functions. Thus, the implicit assumption of this model, that the description of local behaviour by these parameters is without loss of generality, and is only an approximation of global behaviour, is lost. Then, the strong assumption that global behaviour is described by the global utility function as used for calibration, must be made.

Care must be taken with the interpretation of the magnitude of virtual tax rates. A high rate does not automatically imply a heavy distortion, as this rate is inversely related to the elasticities of substitution. These elasticities depend upon the level of disaggregation of goods and sectors. For example, disaggregation of rationed goods, combined with high elasticities of substitution between the composites and an uneven distribution of the rationing levels among these composites might result in lower virtual tax rates.

3. The level of virtual labour taxes in the Netherlands, 1979-1984

In table 1 a summary is given of the Dutch private household expenditures for necessities, luxuries, savings, labour and capital services for the years 1979-1984. As before, negative amounts refer to supply of goods. The amounts in part A refer to net payments and in part B to tax payments by the private household. The figures are from the so called 'Total Accounts'. Total Accounts are related to the widely used Social Accounting Matrices (SAM's), which are composed for many less developed countries (see Pyatt and Round, 1985, and see CBS, 1987, 1989). Total Accounts give a summary of demands and supplies by household and firm sectors expressed as expenditure flows, together with the tax payments by these sectors. Tax payments are divided into lump-sum taxes and commodity taxes. Lump-sum taxes are treated as negative 'lump-sum transfers', and are therefore subtracted from e.g. social security transfers, while the level of transaction taxes depends on the amounts paid (ad valorem taxes, e.g. VAT and labour taxes) or on the quantity transferred (excise taxes). The sum of lump-sum income and all negative demands (= supplies) is called the total income of a household. Here, the method to construct a 'Total Accounts' as given in Keller (1980) is used. The latter method has been programmed into a Lotus 123-worksheet, which needs as input some figures from the yearly published National Accounts (see CBS 1981-1986).

In table 1 part C gives the net prices paid by the private household for each good, relative to the market prices, which are normalized to be 1. In part D the average number of unemployed and the total employment are given for these years. If we suppose that the number of unemployed is an exact measure for excess demand on the labour market, if we suppose other markets were in equilibrium each year, and if we assume a particular utility function to prevail with a particular parameter set, partially fixed for all years and partially changing between years, we can determine the virtual price level on the labour market for each year by using the calibration method proposed in section 2.

Table 1 *The Dutch private household sector 1979-1984*

Good	Year 1979	1980	1981	1982	1983	1984
A. Expenditures and Lump-sum income ($*10^6$ *Glds.)*						
1. Consumption	194040	207500	215280	224060	232170	239160
2. Savings	30590	24830	29570	39210	42660	49730
3. Labour	-103346	-107523	-108377	-108586	-102019	-104282
4. Capital services	-26294	-21807	-25263	-33974	-45841	-58638
Lumpsum income	94990	103000	111210	120710	126970	125970
B. Taxes paid ($*10^6$ *Glds.)*						
1. Consumption	26504	28700	29198	29718	31349	33443
2. Savings	-4080	-7300	-8130	-8300	-7690	-8680
3. Labour	65810	71779	74805	81153	88135	86693
4. Capital services	2590	2541	2575	2777	2895	2987
C. Relative prices paid						
1. Consumption	1.158	1.161	1.157	1.153	1.156	1.163
2. Savings	0.882	0.773	0.784	0.825	0.847	0.851
3. Labour	0.611	0.600	0.592	0.572	0.537	0.556
4. Capital services	0.910	0.896	0.908	0.924	0.941	0.951
*D. Employment and Unemployment (*1000)*[1]						
Total Employment	4773	4807	4736	4619	4513	4528
Registered Unemployment	281	325	480	655	801	822

[1]Source : CEP, 1988.

We suppose the household to behave according to a generalized multi-level CES utility function (see, Keller 1976). Its substitution structure is given in figure 2. Notice that the capital services are excluded from the utility tree as its supply is fixed.

From table 2 it can be seen that between 1979 and 1983 the real and virtual labour taxes have risen. As unemployment has risen rapidly, the total amount of unemployment benefits has risen too. These social benefits are mainly financed by labour taxation. Therefore, labour taxes paid by the private household have risen too. If it is true that the labour prices paid by the demanders of labour have remain stable, then it can be concluded that the government has acted quite rationally by imposing these labour tax increases to finance the social security benefits. In a sense, the partial imposition of virtual taxes has not disturbed the general equilibrium at all. Thus, the government has used the freedom given by the implicit virtual tax level due to unemployment to finance a social benefit policy of income redistribution without affecting behaviour of the private household and thus without affecting general equilibrium.

Figure 2 *The private household utility tree for 1979-1984*

```
                                    u                level ℓ =
                                    ↑
                            σ = 1   o -------------------- 2

                     σ = 0.1   o ------------------------- 1

                          o --------- o --------- o -------- 0
good n =                  1           3           2
                    consumption     labour      savings
```

Table 2 *Excess demand, real and virtual tax on and virtual price for labour*

	Year					
	1979	1980	1981	1982	1983	1984
Excess demand (%)	5.89	6.76	10.14	14.18	17.68	18.15
Real labour tax[1]	0.389	0.400	0.408	0.428	0.463	0.454
Virtual labour tax[2]	0.469	0.498	0.568	0.617	0.637	0.637
Virtual labour price	0.324	0.301	0.256	0.219	0.195	0.198

[1] with respect to the market wage
[2] with respect to the net wage

From table 3 it can be seen that the market price for labour has remained quite stable, if one accounts for the 'general' decrease of the value of money by inflation. Further, it can be concluded that the labour prices paid by the public sector and the firms have remained relatively stable, if the inflation rate has been accounted for. Only, if the labour unions would have succeeded to force market wage increases in order to compensate for the tax increases, imposed on households, the real equilibrium would have been affected. This has not happened.

Table 3 *Labour price, inflation, labour productivity increase and taxation rates on labour per sector for 1978-1984*

	Year					
	1979	1980	1981	1982	1983	1984
Market Price (Glds)[1]	35440	37300	38679	41077	41967	42176
Id., Increase (%)		5.2	3.7	6.2	2.2	0.5
Inflation (%)[2]		6.9	6.3	5.3	2.7	2.0
Labour Productivity (%)[2]		0.6	3.3	2.7	6.8	5.8
Labour tax (% of market price) paid by:						
Firm sector	21.0	21.3	21.4	20.8	21.6	21.2
Public sector	39.1	39.3	38.8	36.9	40.8	39.7
Private sector	38.9	40.0	40.8	42.8	46.3	45.4

[1] This figure is computed as (net labour income + taxes on labour paid by the private household)/total employment. See tables 1 and 2.
[2] Source: CEP, 1988.

Therefore, the claim of the government in those years that the Dutch social security system was a burden on the economy, and that it, therefore, aggravated unemployment can be partially invalidated. Of course, the effect on economic growth cannot be determined in this model. But, for the real and virtual taxation levels given, the general equilibria have not been affected, as a decrease of the social security benefit levels and the accompanying taxes would have induced an opposite effect on the virtual tax level and accompanying virtual lump-sum benefit. Of course, this is only true at this very high level of aggregation, where distribution aspects of the tax-system between various households is abstracted from.

In section 3 two questions will be addressed in a an application to the 114-sector model of the Netherlands in 1981. First, it will be investigated, how the introduction of virtual taxes at the initial equilibrium affects the outcomes for real tax changes. Second, the question will be addressed how price rigidities supported by virtual tax changes influence the model outcomes.

4 An application to the 114-sector model

In this section, the virtual-tax idea to represent rationing is applied to the 114-sector version of the Keller model (Keller, 1980) for the Dutch economy in 1981. The model has been used too in Keller e.a. (1988), Van de Stadt e.a. (1989), and Zeelenberg e.a. (1989) for evaluation of recent tax proposals of the Dutch government. Here, the model is applied to the Netherlands' economy in 1981, distinguishing 114 sectors and 65 goods. We will carry out a simulation of a subsidy on low-paid labour services. In recent years, such a subsidy has been proposed by several authors as a means of reducing unemployment among low-paid workers, which, as we will see below, is about twice as large as unemployment amomg other employees. Most authors propose a subsidy (e.g. a decrease of social contributions) that is highest for wages equal to the minimum wage and gradually decreases for wages above the minimum wage.

The 114 sectors consist of 57 firm sectors (including the production of public goods by the government), the capital goods sector, the public sector, the rest of the world, 52 private household sectors, and 2 fictitious household sectors that administer accumulated corporate savings and pension and life insurance wealth of households. The data are derived from the National accounts (CBS, 1986), the input-output tables (CBS, 1984), the Socio-economic accounts (CBS, 1988), and the Income statistics (CBS, 1985, 1986b); the data, together with several simulations, will be published in CBS (1990).

The firm sectors are identical to the industries in the input-output table. The private household sectors are identical to the household sectors of the Socio-economic accounts: a cross-classification of households by socio-economic status (private employees, public employees, pensioners, recipients of other social benefits, and self-employed), household size (1 person, 2 persons, and 3 or more persons), and income class (25%-groups of net household income); households of self-employed are not further divided according to size or income class. The number of households varies considerably across sectors, from 2,000 to 500,000; 8 household sectors make up for 50 per cent of all households.

The 65 goods consist of the 57 products of the firm sectors, one type of capital goods, 2 types of imported products (competing and complementary imports), 4 types of labour services (low-paid, medium-paid, and high-paid labour services of employees, and labour services of self-employed), and capital services. Low-paid labour services correspond to gross wages up to 1.2 times the minimum wage, medium-paid labour services to gross wages from 1.2 until 1.7 times the minimum-wage, and high-paid labour services to gross wages above 1.7 times the minimum-wage.

The price and income elasticities are derived from detailed econometric studies of household and firm behavior. Most price elasticities of households are computed from the time-series study by Keller and Van Driel (1985), the income elasticities from the cross-section study by Van Driel (1987), and the price elasticities of firms from the time-series analysis by Donkers and Kreijger (1985). The elasticities are specified by means of nested CES functions.

The aggregate income elasticity of labour supply is about -0.2. Supply of capital services is assumed to be constant, so that its income elasticity and its partial elasticity of substitution with other goods are 0. The aggregate income elasticity of capital goods (savings) is about 2, and the partial elasticity of substitution between capital goods on the one hand and products and labour on the other hand is equal to 1; this implies that the interest elasticity of savings is about 0.2. The partial elasticity of substitution between supply of employee labour and supply of self-employed labour is equal to 1, and the partial elasticity between the 3 types of labour services of employees is also equal to 1. For most household sectors this implies an own-price elasticity of employee-labour supply of about 0.2 and an own-price elasticity of self-employed-labour supply of about 0.1.

The partial elasticity of substitution between demand for employee labour and demand for self-employed labour is equal to 0, and the partial elasticity between the 3 types of labour services of employees is equal to 1. For most firm sectors this implies an own-price elasticity of employee-labour demand of about -0.9 and an own-price elasticity of self-employed-labour demand of about -0.2. The partial elasticity of substitution between demand for capital and demand for labour is for most manufacturing sectors equal to 0.2 and for most services sectors between 0.5 and 1.

In order to compute the virtual taxes we need data on excess demand for labour, differentiated by household sector. We measure the percentage of excess demand by unemployment compensation (divided by the replacement ratio). This gives only a rough measure of excess demand, but we think that the relative differences between household sectors are sufficiently well measured. Unemployment compensation consists of unemployment insurance benefits and social assistance grants to unemployed. Due to lack of data, unfunded unemployment compensation to public employees (about 7 per cent of total unemployment compensation) has been excluded.

In absolute terms (measured by the amount of unemployment compensation), most of the excess demand occurs in the households of private employees and recipients of other social benefits (respectively 30 and 60 per cent of total excess demand); this is partly a consequence of our excluding unemployment compensation to public employees. Excess demand is almost evenly spread across income classes, and increases with household size.

In relative terms (measured by unemployment compensation as a ratio of gross wages plus unemployment compensation), excess demand is about 10 per cent for low-paid labour services and 5 per cent for medium- and high-paid labour services. It is extremely high, more than 50 per cent, in households of social-benefit recipients; but labour supply (measured in gross wage) in these households is only 2 per cent of total labour supply, so that for the economy as a whole this high excess demand does not carry much weight. Since almost all households in the lowest income class are either pensioneers or recipients of other social benefits, relative excess demand is also high, more than 25 per cent; but these households receive only 3 per cent of gross wages of all households. Excess demand is also high, about 10 per cent, for single-person households, who receive 6 per cent of total gross wages (most of these households consist of single-person pensioneers).

As pointed out in section 3, partially unemployment benefits are 'realized' virtual taxes. The benefits are financed by premiums paid on actual labour supply of households, labour demand of firms, and from general taxes. Therefore, the unemployment insurance scheme redistributes income from employed households towards unemployed ones. If one would adapt the Total Accounts by excluding these benefits, as far as these are not due to income redistribution, and the accompanying premiums, it is possible to compute a second set of virtual tax levels. If one compares this set with the set derived from the original Total Accounts, it is possible to determine to which level the unemployment benefits are 'realized' virtual taxes, which do not affect general equilibrium. Notice, that the remaining income redistributing unemployment benefits may be ascribed to the hard core of those who are unemployed and have small chances to find work. Thus, without taking into account these unemployment benefits, virtual taxation totally accounts for unemployment and includes these benefits.

We have simulated a subsidy on the use of low-paid labour services by firms. The results refer to a uniform reduction of 1 per cent of the market wage of low-paid labour services, which amounts to a subsidiy of dfl. 300 million. The reduction is assumed to be uniform, i.e. the same for all firms and for all employees in the low-paid category.

It is assumed that the lump-sum distribution is not affected by changes in the virtual tax levels. Unemployment compensation is a component of lump-sum transfers to private households. If owing to the subsidy, unemployment decreases, unemployment compensation and lump-sum transfers should then also decrease. This decrease is one possible source of finance for the subsidy. Also, because social security contributions for unemployment are earmarked taxes, they will change when unemployment changes. However, we have not taken into account the changes in unemployment compensation and social contributions that result from the subsidy, but have assumed that the subsidy is financed by a reduction of government outlay, evenly spread over expenditure on public goods, government savings, and unrequited transfers to households. Since employee labour is an important input in the production of public goods, a change in the demand for public goods may have considerable effects on the labour markets, and since transfers are a large component of household disposable income, a change in transfers may have considerable effects on real income. Therefore the results of the simulation should be regarded more as an example of the possibilities of the model than as relevant for actual policy making.

In carrying out the simulation, we have rationed household supply of the 3 types of labour services of employees (low paid, medium paid, and high paid); labour services of self-employed have not been rationed.

From table 4 it can be seen that a virtual tax indeed increases the price for the labour type on which it is raised. Owing to countervailing substitution effects at both the household and firm sides, trade is shifted to the other two labour markets, with a resulting decrease of the market prices.

Table 4 *Virtual tax price effects, percentage of market prices*

	Virtual tax on labour		
Labour type	low	middle	high
Low-paid labour	0.1820	-0.0065	-0.0049
Medium-paid Labour	-0.0010	0.1548	-0.0094
High-paid labour	-0.0013	-0.0083	0.1674

In both cases with and without price-rigidities, the subsidy decreases the labour-cost for low-paid labour of firms. This increases the demand for low-paid labour, as the price-elasticities of the firms for this type of labour are positive. In the case of rationing demand raises 0.8% as no 'tax'-shift can occur due to the fixed market wage, while in the case of price-flexibility demand raises by only 0.2%. To keep market prices constant, the virtual tax on low-paid labour must decrease. As low-paid labour becomes cheaper, demand for medium-paid and high-paid labour will be lowered. In case of rationing, this cannot be offset partially by a market price decrease, as those are fixed too. Therefore, the virtual taxes on the medium-paid, and high-paid labour must increase slightly. Ultimately, the demand for low-paid labour services increases by 0.8 per cent and the demand for medium- and high-paid labour services decreases by 0.1 per cent; in terms of jobs this means a increase of 15,000 low-paid jobs, and a decrease of 2,000 medium-paid and 2,000 high-paid jobs, so that the total number of jobs increases by about 10,000 jobs. Because the net increase in jobs is smaller than the increase in low-paid jobs, the subsidy per extra job (dfl. 28,000) is higher than the average gross wage of low-paid employees (dfl. 25,000). With constant labour supply, a subsidy of about 5 per cent (dfl. 1,500 million) would eliminate the difference in relative excess demand between low-paid labour services and medium- and high-paid labour services; relative excess demand would then be about 6 per cent.

Figure 5 *Comparison of the changes in market prices with and without rationing (perc.)*

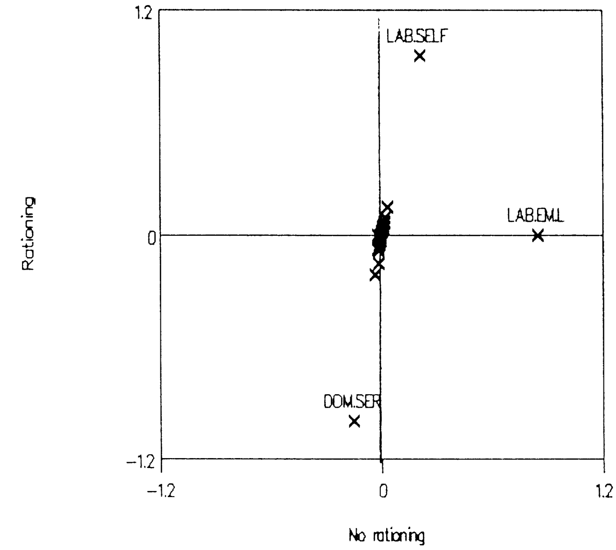
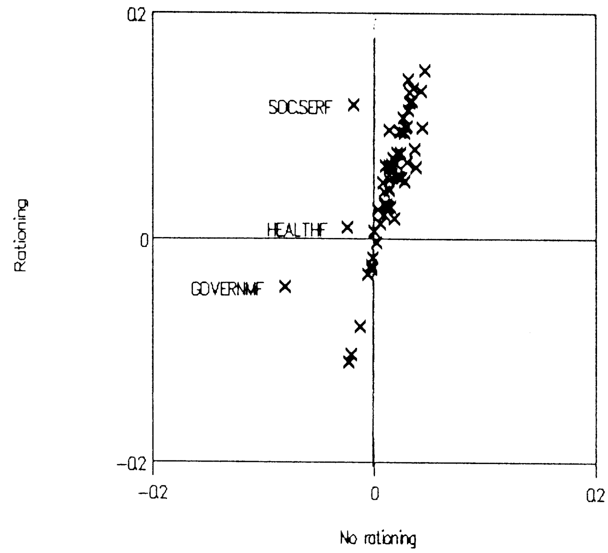


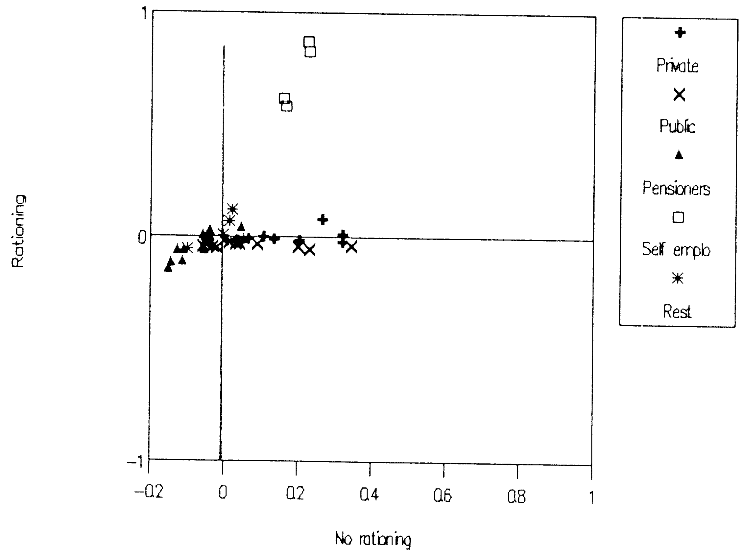
Figure 6 *Comparison of the changes of output levels with and without rationing (perc.)*



If one compares the changes in market prices without and with rationing, it turns out that the market price changes under virtual taxation are slightly larger, but remain less than 0.1% (see figure 5). The changes are magnified by roughly the same factor, except for the self-employed labour services, low-paid labour services, domestic services, agriculture and social services. The market price change of low-paid labour services is kept constant by definition in stead of being allowed to increase with 0.85%. Domestic services are produced by a firm using one single input: low-paid labour services, of which the net price decreases with 1 percent under rationing. This implies a 1 % market-price decrease for domestic services too, which is not accompanied by an increase in demand for this type of services owing to a low price elasticity. The same line of reasoning can be followed for the price of and demand for social services, explaining the relatively large decrease of the market price. The market-price for agricultural products increases by 0.15%, mainly because of the increase in the price of self-employed labour, which is an important input in agriculture. This priceincreases 0.95% under rationing instead of 0.21%. The relative decrease of total labour cost of firms can be translated neither into a larger trade of labour due to a relatively inflexible supply, nor into an increase of the prices for all types of labour together. Only, the demand for self-employed labour can be offset by a market-price increase for self-employed labour to retain market equilibrium. However, trade changes by less than 0.05%.

From figure 6, it is clear that under rationing the output changes of almost all firms are magnified by a constant factor with respect to those under absence of rationing. The output change of the government firm is almost not affected because this change is dependent on the real income change of the public household, which is the sole demander of this good. This change is quite equal under both conditions, as the total tax yield change is not affected much by introduction of rationing (0.09% in stead of 0.08%). The firm which produces health services shows a slight increase in supply in stead of slight decrease. This is due to a larger output price change. The same holds for the firm producing social security services. Both firms have a relatively large demand for low-paid labour.

Figure 7 *Comparison of real income changes with and without rationing (perc. of own income)*



If one compares the real income changes of households (see figure 7), it turns out that the real income changes decrease for the households of public and private employees, but increase sharply for the households of self-employed under rationing. The latter is due to the large increase of the market price for self-employed labour, which has a large income effect. For the private and public employee households, the opposite occurs, as the market price of low-paid labour stays fixed instead of being increased by the subsidy. As the pensioner households do not supply much labour, their real income is affected mainly by the change in lump-sum income, which is proportional to the change in the total tax yield. For most private households the decrease in transfers (the financing of the subsidy) has a larger effect on real income than the increase in employment; thus real income decreases, in particular for households of social-benefits recipients. If we had taken into account that unemployment compensation decreases if employment increases, the decrease in real income would be more evenly spread: real income of households of private employees would decrease more and real income of households of social-benefits recipients would decrease less. Only for some households whose labour supply consists of low-paid labour services, real income increases (private employees/2 and 3 or more persons/1st income class).

As already noticed, total tax revenue decreases about respectively 0.08 and 0.09 per cent without and with rationing, which is somewhat less than the initial decrease of 0.1 per cent. This is mainly caused by the increase in employment, which leads to a larger revenue of the income tax and social-security contributions.

To end this section, it can be concluded that the effects under rationing differ from the effects without rationing. The main changes are due to the fixed market prices for low-, medium and high-paid labour. The low-paid labour cost decrease of firms under rationing is multiplied by a factor 5 when compared to the outcome without rationing. The resulting market-price changes for other goods are multiplied by a similar factor too. Because without rationing the price of low-paid labour increases, real income of households of employees also increase, except in households that do not supply low-paid labour (e.g. single persons/3rd and 4th income class). Thus except for real income changes, the direction of the effects is the same with or without rationing, although the size of the effects may differ.

6. Concluding remarks

In this paper a theoretically sound method to represent rationing by equivalent virtual taxes and a calibration-method based on this equivalence has been presented. First, the level of virtual taxation for the Netherlands in 1979-1984 was studied. Next, the methods were applied to the 114-sector variant of the Keller-model for the Netherlands in 1981. The effects of a low-paid-labour subsidy were studied. It turned out, that the welfare changes are affected by the introduction of labour rationing. Production in almost all sectors increases. The outcomes with rationing showed stronger effects of the subsidy on market prices and output levels, while the welfare effects were much more negative for the employee households.

Literature

Benassy, J.P., 1975, **Neo-Keynesian disequilibrium theory in a monetary economy**, Review of Economic Studies 42, 503-523.

Centraal Bureau voor de Statistiek, 1982-1987, **Nationale rekeningen**, (National accounts), (Staatsuitgeverij, 's Gravenhage).

Centraal Bureau voor de Statistiek, 1984, **De productie structuur van de Nederlandse volkshuishouding, deel 12: input-output tabellen en input-output coefficienten 1981**, (The production structure of the Netherlands' economy. Part 12: Input-output tables and input-output coefficients 1981), (Staatsuitgeverij, 's Gravenhage).

Centraal Bureau voor de Statistiek, 1985, **Personele inkomensverdeling 1981: individuen**, (Distribution of personal income 1981: individuals). (Staatsuitgeverij, 's Gravenhage).

Centraal Bureau voor de Statistiek, 1986b, **Personele inkomensverdeling 1981: huishoudens**, (Distribution of personal income 1981: households), (Staatsuitgeverij, 's Gravenhage).

Centraal Bureau voor de Statistiek, 1988, **Sociaal-economische rekeningen: methodebeschrijving en toepassing in 1981**, (Socio-economic accounts: methodology and application for 1981), (Staatsuitgeverij, 's Gravenhage).

Centraal Bureau voor de Statistiek, 1990 (forthcoming), Tax incidence in the Netherlands, (SDU uitgeverij, 's Gravenhage).

Centraal Plan Bureau, 1988, **Centraal Economisch Plan 1988** (SDU Uitgeverij, 's Gravenhage).

Cornielje, O.J.C, 1984, **From local to global in applied general equilibrium modelling: the multi-level CES case**, IAWE discussion paper 135 (Free University of Amsterdam, Amsterdam).

Cornielje, O.J.C., 1985, **Calibration in applied general equilibrium models, the case of rationed households**, Economics Letters 19, 211-215.

Cornielje, O.J.C., 1989, **A Time-series of Total Accounts for The Netherlands 1978-1984**, FEWEC Research memorandum 1989-1 (Free University of Amsterdam, Amsterdam)

Cornielje, O.J.C., 1990 (forthcoming), **Rationing and capital mobility in applied general equilibrium models**, PhD thesis.

Cornielje, O.J.C. and W.J. Keller, 1983, **Simulating rigidities and rationing by taxes in general equilibrium models**, IAWE discussion paper 109 (Free University of Amsterdam, Amsterdam).

Cornielje, O.J.C. and G. van der Laan, 1986, **The computation of quantity constrained equilibria by virtual taxes** , Economics Letters 22, 1-6.

Donkers, H.W.J. and R.G. Kreijger, 1985, **Estimating sectoral cost functions**, internal report (Centraal Bureau voor de Statistiek, Voorburg).

Grais, W., J. de Melo and S. Urata, 1986, **A general equilibrium estimation of the effects of reductions in tariffs and quantitative restrictions in Turkey in 1978**, in Srinivaran, T.N. and J. Whalley (eds.), General Equilibrium Trade Policy Modeling (The MIT Press, Cambridge USA).

Guesnerie, R. and K.W. Roberts, 1984, **Effective policy tools and quantity controls** , Econometrica 52, 59-86.

Hicks, J.R., 1946, **Value and Capital** (Clarendon Press, Oxford).

Johansen, L., 1960, **A multi-sectoral study of economic growth** (North Holland, Amsterdam).

Keller, W.J., 1976, **A nested CES-type utility function and its demand and price index functions**, European Economic Review 7, 175-186.

Keller, W.J., 1980, **Tax incidence, a general equilibrium approach**, (North Holland, Amsterdam).

Keller, W.J. and J. Van Driel, 1985, **Differentiable consumer demand systems**, European Economic Review 27, 375-390.

Keller, W.J., C. Zeelenberg, R.D. Huigen and P. Kooiman, 1988, **Effecten van belastingheffing: een toegepaste algemeen-evenwichtsanalyse** (Effects of taxation: an applied general equilibrium analysis), in Preadviezen voor de Vereniging voor de Staathuishoudkunde (Stenferd Kroese, Leiden).

King, B.B., 1985, **What is a SAM?**, in Pyatt, G. and J.I. Round, **Social accounting matrices, a basis for planning** (The World Bank, Washington D.C.).

Kooiman, P., 1982, **Using business survey data in empirical disequilibrium models**, ICERD discussion paper (London School of Economics, London).

Laan, G. van der, 1984, **Supply-constrained fixed price equilibria in monetary economies**, Journal of Mathematical Economics 13, 171-187.

Malinvaud, E., 1978, **The theory of Unemployment reconsidered** (Basil Blackwell, Oxford).

Mansur, A. and J. Whalley, 1984, **Numerical estimation of applied general equilibrium models: Estimation, calibration and data**, in: H.E. Scarf and J.B. Shoven, eds., Applied general equilibrium analysis (Cambridge University Press, Cambridge).

Neary, J.P. and K.W. Roberts, 1980, **The theory of household behaviour under rationing**, European Economic Review 13, 25-42.

Pyatt, G. and J.I. Round, 1985, **Social accounting matrices, a basis for planning** (The World Bank, Washington D.C.).

Rothbard, E., 1941, **The measurement of changes in real income under conditions of rationing**, Review of Economic Studies 8, p.25-42.

Tobin, J. and H.S. Houthakker, 1951, **The effects of rationing on demand elasticities**, Review of Economic Studies 18, 140-153.

Van Driel, J., 1987, **Inkomenselasticiteiten voor verschillende huishoudtypen geschat uit het Doorlopend Budgetonderzoek 1981**, (Income elasticities for varying household-types estimated from the continuous budget survey 1981), (Centraal Bureau voor de Statistiek, Voorburg).

Van de Stadt, H, R.D. Huigen and C. Zeelenberg, 1989, **De Oort-voorstellen: simulaties met een algemeen evenwichtsmodel** (The Oort-proposals: simulations with a general equilibrium model), unpublished working paper (Centraal Bureau voor de Statistiek, Voorburg).

Zeelenberg, C., R.D. Huigen, and P. Kooiman, 1989, **Total Accounts: an accounting system for applied general equilibrium analysis**, unpublished working paper 11122-89-M1 (Centraal Bureau voor de Statistiek, Voorburg).

Zeelenberg, C., R.D. Huigen, and H. Van der Stadt, 1989, **Gevolgen van belastingvoorstellen in het Nationaal Milieubeleidsplan**, (Effects of the tax proposals in the National environmental policy plan), unpublished working paper 10430-89-M1-1 (Centraal Bureau voor de Statistiek, Voorburg).

Appendix A. Changes in the Keller model in order to cope with rationing of households due to price-rigidities

In this appendix we summarize the necessary adaptions in the Keller model (see Keller, 1980, chapter 12). We refer to notation and equations in Keller (1980).

The first question arises how to compute the price and income elasticities of the households. It is clear that besides the information about the cost shares, marginal cost shares, the knot-structure and substitution elasticities of the underlying CES-utility-functions some information must be added about the virtual tax rates and virtual lump-sum income for each household. This information can be given in either absolute amounts or relative figures with respect to real income. Then before the computation of the price and income elasticities may take place income shares are adapted according to

$$c^*_n = c_n + \tau^*_n \quad , n = 1,\dots,N \tag{A1}$$

where these shares are expressed relative to total, non virtual income ν. The tax shares τ^*_n are defined as λ^*_n/ν, with λ^*_n as given in section 2. If one takes ν as denominator, all income-concepts remain equal with and without rationing and virtual taxation. Notice, that the sum of budget-shares does not equal λ/ν as usual (see Keller, chapter 4) but to $(\lambda+\Sigma\lambda^*_n)/\nu$. Still, the various adding-up restrictions hold, but with the virtual cost shares c^* instead of the real cost shares c :

$$c^{*\prime} N_H + c^{*\prime} = 0 \quad , \tag{A2}$$

$$c^{*\prime} n_H = 1 \quad , \tag{A3}$$

$$N_H \iota + \varphi^* n_H = 0 \quad , \tag{A4}$$

where $\varphi^* = \lambda^*/\nu$. Next one can compute aggregate household behaviour. First, use instead of equation (12.1) for each household i = 1,...,I:

$$\tilde{q}^i_H = \left(I - n^i_H \tau^{*i'}_H\right)^{-1}\left(N^i_H + n^i_H \tau^{*i'}_H\right)\tilde{p}^i_H + \left(I - n^i_H \tau^{*i'}_H\right)^{-1} N^{*i}_H \tilde{t}^{*i}_H + \left(I - n^i_H \tau^{*i'}_H\right)^{-1} n^i_H \tilde{\lambda}^i$$

$$= \bar{N}^i_H \tilde{p}^i_H + \bar{N}^{*i}_H \tilde{t}^{*i}_H + \bar{n}^i_H \tilde{\lambda}^i \quad . \tag{A5}$$

Thus the uncompensated price elasticities N^i_H are computed as an intermediate result which is used to compute the price elasticities under rationing. It is simple Linear Algebra to show that

$$\left(I - n^i_H \tau^{*i'}_H\right)^{-1} = I + \frac{n^i_H \tau^{*i'}_H}{1 - \tau^{*i'}_H n^i_H} \quad . \tag{A6}$$

Using equations (A1)-(A6), it simple to prove that the next adding-up restrictions hold

$$c^{i'}_H \bar{N}^i_H + c^{i'}_H = 0 \quad , \tag{A7}$$

$$c^{i'}_H \bar{n}^i_H = 1 \quad , \tag{A8}$$

$$\bar{N}^i_H \iota + \varphi^i \bar{n}^i_H = 0 \quad , \tag{A9}$$

where $\varphi^i = \lambda^i / \nu^i$. The resulting behaviour as given by these elasticity matrices still fulfill the basic restrictions of consumer behaviour due to the real budget constraint. These constraints are necessary conditions to arrive at a meaningful solution of the model.

Next, a similar equation like (12.7) can be derived using equation (A5):

$$\tilde{q}_H = N_{HM}\tilde{p}_M + N_{HT}\tilde{t} + N^*_{HT}\tilde{t}^*_H + n\tilde{\rho} \tag{A10}$$

where the additional term $N^*_{HT}\tilde{t}^*_H$ is defined by

$$N^*_{HT}\tilde{t}^*_H = \sum_{i=1}^{I} \hat{a}^i_H \bar{N}^{*i}_H \Pi^{*i} \tilde{t}^*_H \quad . \tag{A11}$$

Suppose T^* virtual taxes are distinguished. Then the $N\mathrm{x}T^*$ tax-flexibilitymatrix Π^{*i} is defined by

$$\Pi^{*i}\tilde{t}^{*}_{H} = \tilde{t}^{*i}_{H} \quad , \qquad i = 1,\ldots,I \tag{A12}$$

These 'virtual tax flexibility matrices' makes it possible to introduce various rationing schemes with respect to the goods and households affected. The final equation of the Keller model to solve becomes

$$\begin{bmatrix} M_{FF}N_{HM}-M_{FM} & M_{FF}n \\ w' & 0 \end{bmatrix} \begin{bmatrix} \tilde{p}_M \\ \tilde{\rho} \end{bmatrix} = \begin{bmatrix} M_{FT}-M_{FF}N_{HT} & -M_{FF}N^{*}_{HT} \\ 0 & \end{bmatrix} \begin{bmatrix} \tilde{t} \\ \tilde{t}^{*} \end{bmatrix} \tag{A13}$$

From this equation all price changes, tax yield changes, etc. can be derived due to 1% changes of real and virtual taxes.

As one supposes that market-price-rigidities are the origin of rationing changes of real taxes must be accompanied by virtual tax changes $\tilde{t}^{*}$ to keep $\tilde{p}_{Mn} = 0$ for some goods $n = 1,\ldots,N$. Using a partitioning scheme on the solution of (A13) we derive that

$$\begin{bmatrix} \tilde{p}_{MX} \\ \tilde{p}_{MY} \\ \tilde{\rho} \end{bmatrix} = \begin{bmatrix} A_{XT} & A^{*}_{XT} \\ A_{YT} & A^{*}_{YT} \\ A_{\rho} & A^{*}_{\rho} \end{bmatrix} \begin{pmatrix} \tilde{t} \\ \tilde{t}^{*} \end{pmatrix} \quad . \tag{A14}$$

As $\tilde{p}_{MY}=0$ must hold, from (A14) the necessary tax changes $\tilde{t}^{*}$ can be computed from

$$\tilde{t}^{*} = -A^{*-1}_{YT}A_{YT}\tilde{t} \quad , \tag{A15}$$

This is only possible of the inverse of A^{*}_{YT} is a $T^{*}xT^{*}$-matrix of full rank. E.g., only one virtual tax-instrument may be defined for each rationed good affecting a household with flexible demand for this good and initial non-zero demand. When equation (A15) is substituted into (A14) the final solution of the Keller model under price-rigidities on good Y is derived

$$\begin{bmatrix} \tilde{p}_{MX} \\ \tilde{p}_{MY} \\ \tilde{\rho} \end{bmatrix} = \begin{bmatrix} A_{XT} - A^{*}_{XT} A^{*-1}_{YT} A_{YT} \\ 0 \\ A_{\rho} - A^{*}_{\rho} A^{*-1}_{YT} A_{YT} \end{bmatrix} \tilde{t} \quad . \tag{A16}$$

Using this solution all changes in quantities, welfare and net prices can be computed.

To conclude this appendix it must be noted that the procedure outlined here is only valid for rationing of households, e.g. for unemployment. It presupposes that rationing is due to market price rigidities and the rationing schemes can be determined exogenously. As the concept of market prices is arbitrary, net price rigidities could be introduced by using the equations which relate market-prices and real tax-changes to individual net prices as given by Keller (1980), eqs. (12.3) and (12.4). It results in minor adaptions of the changes outlined in this appendix.

If initially rationing occurs, as said earlier, the benchmark year data-set must be adapted. To summarize, the next computational procedure must be followed

1. Adapt the benchmark year data set according to Cornielje (1985).
2. Check for an initial general equilibrium using the original data-set.
3. Compute the individual price and income elasticity matrices of the households, using the adapted data-set.
4. Next adapt the individual matrices according to equation (A5) if initially rationing occurs.
5. Compute the effects of changes in real and virtual taxes.
6. Compute the necessary virtual tax changes to induce price-rigidities for each real tax instrument. Compute the real price changes under price rigidities by summing the effects of real tax changes and accompanying virtual tax changes.
7. Compute the changes in individual demand etc.. For this only the combined effect of real tax changes and accompanying virtual tax changes on the prices and real tax yield has to be known.